NONLINEAR FLUTTER OF PLATES IN THE TURBULENT BOUNDARY LAYER OF A WEAKLY COMPRESSIBLE FLOW

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UDC 537.86/.87:530.182:533.6.013.42

A study of the steady vibrations of plates in flows of liquid and gas (nonlinear panel flutter) makes it possible to determine the actual level of vibration of the surface in the flow. There has already been a large number of studies of nonlinear plate flutter in uniform supersonic flows (see [1-3], for example). Dowell [4] demonstrated the potential for dynamic chaos in supersonic panel flutter. Solving the analogous problem for subsonic flow requires allowance for the effect of interaction of plate deflection with the boundary layer of this flow.

In the present study, we examine the nonlinear flutter of one plate and a system of two adjacent plates with fastened edges. The plates are under the turbulent boundary layer of a slightly compressible (essentially subsonic) flow. The plates are butt-joined along the flow at the same level and have a flat rigid surface. The density of the stationary medium on the opposite side of the surface is assumed to be negligibly low. We will explore the possibility of the onset of dynamic chaos in such a system. The analysis will be based on the results in [5, 6], which determined the linear response of the mean flow to unimodal harmonic vibration of the plates.

1. Analytic Approximations for Elements of the Matrix of Apparent Additional Elasticities. Following [6], we will examine the vibration of adjacnet plates in a plane-parallel flow whose velocity profile coincides with the profile of longitudinal velocity of the mean flow in the turbulent boundary layer. The deflection of each plate will be approximated by the first mode of the Galerkin expansion used in [6]. Boundary conditions on the edges of the plates and restrictions on their geometric dimensions were also given in [6]. An expression of the following from [6] can be written for the mechanical response of the flow to the harmonic ($\sim e^{-i\omega t}$) vibrations of the plates

$$F_{1,2\omega}^{(0)} = -\rho u_\infty^2 k_0 (G A_{1,2\omega} + G_{1,2} A_{2,1\omega}). \tag{1.1}$$

Here, we have used the same notation as in [6]: $A_{j\omega}$ is the amplitude of the first mode of deflection of the j-th plate; G and $G_{1,2}$ are elements of the matrix of the dimensionless additional elasticities (for the vibrations in the first mode); ρ is the density of the moving fluid; δ is the thickness of the boundary layer; $k_0 = \pi/L_1$ (L_1 is the dimension of the plates along the flow); and the zero superscript denotes the incompressible-fluid approximation.

To describe nonharmonic vibrations of plates with zero initial distrubances in the flow, we will regard (1.1) as a constraint on Laplace transformations of the variables $F_{1,2}^{(0)}(t)$ and $A_{1,2}(t)$*. This can be done by virtue of the definition of the frequency functions G and $G_{1,2}$ as analytic continuations from the contour of integration in the inverse Laplace transform to the real axis of the complex plane ω [5].

For practical realization of transformations from (1.1) to a time representation, we approximate G and $G_{1,2}$ by analytic functions ω. We require that these approximations ensure agreement between the control frequency relations obtained from them and the analogous relations constructed from calculations of G and $G_{1,2}$ within a finite interval of real values ω. We will proceed from the representation of G and $G_{1,2}$ as the sum of the principal part (which coincides with the quasi-potential approximation for these functions — see [6]) and small

*The complex parameter ω is related to the additional Laplace transform parameter p by the equation $p = -i\omega$.

Nizhnii Novgorod. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 2, pp. 58-66, March-April, 1993. Original article submitted August 5, 1991; revision submitted March 20, 1992.

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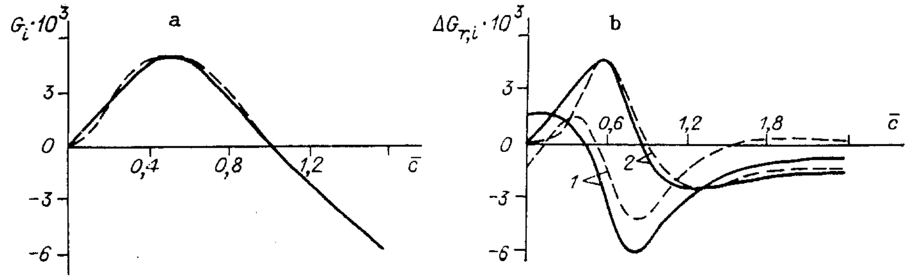

Fig. 1

additional terms reflecting dissipative processes. We write the approximations in the dimensionless variables $\bar{k}_0 = k_0\delta$, $\bar{c} = \omega/k_0 u_\infty$, used in [6]:

$$G = -(a_0\bar{c}^2 + d_0) + \beta[\varphi(\bar{c}) - ig_0\bar{c}]; \tag{1.2}$$

$$\varphi = \frac{iq_0\bar{c} + s_0}{\bar{c}^2 + i\sigma_0\bar{c} - c_0^2}; \tag{1.3}$$

$$G_1 = -(a_1\bar{c}^2 - ib_1\bar{c} + d_1),\ G_2 = -(a_1\bar{c}^2 + ib_1\bar{c} + d_1) - \beta_1\varphi_1(\bar{c}); \tag{1.4}$$

$$\varphi_1 = \frac{iq_1\bar{c} + s_1}{\bar{c}^2 + i\sigma_1\bar{c} - c_1^2}. \tag{1.5}$$

The parameters of the quasi-potential components in (1.2) and (1.4) at $\bar{k}_0 = 1$, $R = 8 \cdot 10^4$ and the plate dimension ratio $L_2/L_1 = 3$ have the form [6] $a_0 = 1.8$, $d_0 = 0.46$, $a_1 = 0.54$, $b_1 = 0.43$, $d_1 = 0.10$. The parameters of the dissipative complements are chosen on the basis of agreement of the functions G_i and $\Delta G_{r,i}$ $(\Delta G = G_1^* - G_2)$ calculated in [6] and their analogs $\beta(\varphi_i - g_0\bar{c})_i$ and $\beta\varphi_{1r,i}$ at the control points. For these values of $\bar{k}_0$ and R we found $\beta = 0.005$, $q_0 = 0.76$, $s_0 = -1.38$, $\sigma_0 = 1.18$, $c_0 = 0.75$, $g_0 = 1.17$, $\beta_1 = 0.0079$, $q_1 = -0.7$, $s_1 = -0.20$, $\sigma_1 = 0.56$, $c_1 = 0.75$. The dashed lines in Fig. 1a and b, show the initial functions G_i and $\Delta G_{r,i}$, while the solid lines show their approximations ($1{:}\Delta G_r$, $2{:}\Delta G_i$). The coefficients β and β_1 in (1.2) and (1.4) were used to normalize the functions $\varphi - ig_0\bar{c}$ and φ_1: we assumed that $\varphi_i - g_0\bar{c} = 1$ and $\varphi_{1i} = 1$ at the points of the first (reckoned from $\bar{c} = 0$) maximum of G_i and ΔG_i, respectively.

The question of the ambiguity of representations (1.2-1.5) should be examined within the framework of the overall problem of determining the analytic transfer function from its real or imaginary parts assigned for real frequencies ω. The transfer function is found unambiguously if a one-two-one correspondence can be established between its real and imaginary parts on the real axis of the complex plane ω. In electrodynamics, the analogous relation for permittivity is known as the Kramers—Kronig relation [7]. It exists when the transfer function describes the relaxational response of the medium. In the absence of a transfer function, this is formally expressed as singularities in the upper half-plane and its approach toward zero at $|\omega| \to \infty$, $\mathrm{Im}\,\omega \geq 0$.

If we assume that the terms $-\beta g_0\bar{c}$ in (1.2) for G_i approximates (within a finite frequency interval) the nondecreasing part of the asymptote G_i for large ω, then the second term $\beta\varphi_i$ can be identified with the imaginary part of the relaxational response of the flow. In this case, Eq. (1.5), giving the required profile of G_i with real ω, unambiguously defines this response in the form of an analytic function of frequency ω. We initially represent the matrix elements $G_{1,2}$ in the form $G_{1,2} = -(a_1\bar{c}^2 \mp b_1\bar{c} + d_1) + z_{1,2}$, where the analytic functions $z_{1,2}(\omega)$ satisfy the condition $z_{1,2} \to 0$ at $|\omega| \to \infty$, $\mathrm{Im}\,\omega \geq 0$ and have no singularities in the upper half-plane. We will express $z_{1,2}$ through the small difference $\Delta G = G_1^* - G_2$, which determines the energy dissipated by the vibrations. To do this, we introduce analytic functions $\Phi_1(\omega)$ and $\Phi_2(\omega)$, which with real ω satisfy the conditions $\Phi_{1r} = \Delta G_r$, $\Phi_{2i} = \Delta G_i$. Constructing the combinations $(G_1 + G_2)_i$ and $(G_1 - G_2)_r$, we obtain the sought expressions: $z_{1,2} = \frac{1}{2}(\pm\Phi_1 - \Phi_2)$. In particular, if ΔG_r and ΔG_i are the real and imaginary parts of the same analytic function $\Phi(\omega)$ ($\Phi_1 = \Phi_2 \equiv \Phi$), we obtain $z_1 = 0$ and $z_2 = -\Phi$. It can be seen from Fig. 1b that this is the situation which is approximately realized in our case. Meanwhile, $\Phi \approx \beta_1\varphi_1$.

The absence of a dissipative component in (1.4) for G_1 indicates that the dissipative constraint is directed downstream (from plate 1 to plate 2). Physically, this is related to convection of the decaying vortical disturbances discussed above. We emphasize that a weak nonreciprocal dissipative relationship exists between the plates against the background of a strong reciprocal conservative relation.*

2. Dynamic Model for Fluctural Vibration of Plates in a Boundary Layer. The response of the flow (1.1) represents the compelling force in the excitation equation for the deflections of the plates in the first mode:

$$\gamma\frac{d^2A_j}{dt^2} + 2r\frac{dA_j}{dt} + (Dk_0^4 - Nk_0^2)A_j + \frac{3D}{h^2}k_0^4A_j^3 = F_j(t),\ j = 1.2 \tag{2.1}$$

(the notation is the same as was used in [6]). The nonlinearity of the response of the flow in the given case can be roughly evaluated as being on the order of $|k_0A_j|^2$. The flexural nonlinearity in (2.1) leads to a limit on the increase of A_j at a level of the same order as the plate thickness h (see Part 5). Thus, for sufficiently thin plates it is interesting to analyze Eq. (2.1) with linear flow response (1.1).

To solve (2.1), we introduce the new dimensionless variables: $\bar{\omega} = \omega/\omega_0$, $\tau = \omega_0 t$, $\bar{A}_{1,2} = A_{1,2}/h$, $V = u_\infty k_0/\omega_0$, $\omega_0 = [(Dk_0^4 - Nk_0^2)/\gamma_0]^{1/2}$ $(\gamma_0 = \gamma + \rho/k_0)$. Proceeding on the basis of Eqs. (1.2-1.5), we find the auxiliary variables $\zeta_{1,2}(\tau)$ and $\eta(\tau)$ by means of the equations $\zeta_{1,2\bar{\omega}} = \varphi\left(\frac{\bar{\omega}}{V}\right)\bar{A}_{1,2\bar{\omega}}$, $\eta_{\bar{\omega}} = \varphi_1\left(\frac{\bar{\omega}}{V}\right)\bar{A}_{1\bar{\omega}}$. Here, the subscript $\bar{\omega}$ denotes Laplace transformation with respect to τ. These relations can be reduced to polynomial form relative to $\bar{\omega}$, which then allows us to change over to differential equations. Along with the "incompressible" component (1.1), the total response F_j should include the additional terms $F_{1,2}^{(a)}$ accounting for sound radiation [6].

First we write the system of equations for one plate (omitting the bars above $A_{1,2}$):

$$m_0\ddot{A}_1 + 2\bar{r}\dot{A}_1 - d_sA_1 + 3\varkappa A_1^3 = \\ = -\alpha_1\beta(V^2\zeta_1 + Vg_0\dot{A}_1) + \frac{\xi}{m_0}(d_s - 9\varkappa A_1^2)\dot{A}_1; \tag{2.2a}$$

$$\ddot{\zeta}_1 + \sigma_0V\dot{\zeta}_1 + V^2c_0^2\zeta_1 = q_0V\dot{A}_1 - s_0V^2A_1. \tag{2.2b}$$

Here, the dots denote derivatives with respect to τ; $\varkappa = (1 - N/k_0^2D)^{-1}$; $\xi = \frac{4}{\pi^2}\alpha_1\frac{L_2\omega}{c_a} \ll 1$ (c_a is the speed of sound); $d_s = \alpha_1 d_0V^2 - 1$; $m_0 = \alpha + \alpha_1 a_0$; $\alpha = \gamma/\gamma_0$; $\alpha_1 = 1 - \alpha$. The contribution of the radiation losses (the terms ~ξ) is written in transformed form in Eq. (2.2a). Its initial representation $\xi\dddot{A}_1$ leads to an equation in which the leading derivative is accompanied by a small parameter, thus allowing for the occurrence of "rapid motions" [10, 11]. The multipole expansion of the pressure field whose first approximation yielded the radiative component of the response of the medium cannot be used for such motions (with the characteristic time ~ξ). To reduce the order of the initial equation, we transformed the term $\xi\ddddot{A}_1$ by inserting into it expressions obtained for $\ddot{A}$ from the same equation after discarding terms of the order ξ^2, $\bar{r}\xi$, $\beta\xi$. This procedure led to Eq. (2.2a), in which the radiation losses are transformed into the effective nonlinear absorption.

*This distinguishes the given model connecting the plates from the model proposed in [8] to describe a conservative unidirectional relationship between vortical structures in a shear flow (also see [9]).

The equations for a pair of adjacent plates are similarly constructed and have the form

$$m_0\ddot{A}_1 + \alpha_1 a_1 \ddot{A}_2 = -2\bar{r}\dot{A}_1 + d_s A_1 + \alpha_1 V b_1 \dot{A}_2 + \\ + \alpha_1 V^2 d_1 A_2 - 3\varkappa A_1^3 - \alpha_1\beta(V^2\zeta_1 + V g_0 \dot{A}_1) + f_a; \tag{2.3}$$

$$m_0\ddot{A}_2 + \alpha_1 a_1 \ddot{A}_1 = -2\bar{r}\dot{A}_2 + d_s A_2 - \alpha_1 V b_1 \dot{A}_1 + \\ + \alpha_1 V^2 d_1 A_1 - 3\varkappa A_2^3 - \alpha_1\beta(V^2\zeta_2 + V g_0 \dot{A}_2) + \alpha_1\beta_1 V^2 \eta + f_a; \tag{2.4}$$

$$f_a = \frac{\xi}{m_0}\left[(d_s - 9\varkappa A_1^2)\dot{A}_1 + (d_s - 9\varkappa A_2^2)\dot{A}_2\right];$$

$$\ddot{\zeta}_{1,2} + \sigma_0 V \dot{\zeta}_{1,2} + V^2 c_0^2 \zeta_{1,2} = g_0 V \dot{A}_{1,2} - s_0 V^2 A_{1,2}; \tag{2.5}$$

$$\ddot{\eta} + \sigma_1 V \dot{\eta} + V^2 c_1^2 \eta = g_1 V \dot{A}_1 - s_1 V^2 A_1. \tag{2.6}$$

In determining the term for the radiation losses f_a in (2.2-2.3), along with the above-indicated second-order quantities we discarded terms on the order of the product ξ and the coupling factors a_1, b_1, d_1, β_1. The coefficient κ can be excluded from the resulting equations by changing over to the new variables $u_{j\,\text{new}} = u_j\sqrt{\kappa}$ $(u_j = A_{1,2},\ \zeta_{1,2},\ \eta)$. Having in mind such a substitution of variables, without loss of generality we can put $\kappa = 1$ in (2.2-2.6).

We will henceforth use the response parameters indicated in Part 1 in the numerical solution of systems (2.2) and (2.3-2.6). We also assume that $\alpha = 0.2$ ($\alpha_1 = 0.8$), which corresponds to dominance of the additional mass over the intrinsic mass of the plate [5, 6]. The change in dimensionless flow velocity V with the other parameters in (1.2-1.5) fixed is related to the assumption that these parameters depend little on the Reynolds number. Such an approximation is satisfactory when V changes by a factor of 1.5-2.

3. Dynamics of a Conservative System. The qualitative features of the solution of systems (2.2) and (2.3-2.6) can be determined by examining the solutions of the conservative problem obtained from (2.2-2.6) with $\bar{r}$, ξ, β, $\beta_1 = 0$. In this case, the Duffing equation [12-15] follows from (2.2a) for one plate

$$m_0\ddot{A}_1 - d_s A_1 + 3A_1^3 = 0. \tag{3.1}$$

Phase portrait (3.1) has the characteristic form of "points" in the plane A_1, $\dot{A}_1$ at $d_s > 0$ (velocity V is greater than the divergence threshold $V_c = 1/\sqrt{\alpha_1 d_0}$). Energy is conserved on each phase trajectory

$$H = \frac{1}{2} m_0 \dot{A}_1^2 - \frac{1}{2} d_s A_1^2 + \frac{3}{4} A_1^4 = \text{const}. \tag{3.2}$$

The quadratic terms in (3.2) determine the energy of infinitesimal deviations from the neutral equilibrium state. At $d_s > 0$, potential energy (the term $\sim A_1^2$) is negative, while kinetic energy (the term $\sim \dot{A}_1^2$) is always positive. It is because of the different signs of these components that $|A_1|$ and $|\dot{A}_1|$ can increase without limit (within the framework of the linear model) while total energy remains constant. The onset of instability is indicated by the infinite character of the lines of constant energy in the neighborhood of the neutral equilibrium state [the saddle point in phase portrait (3.1)]. The appearance of negative potential energy is connected with the nonequilibrium character of the system and is a formal expression of the possibility of reversible transformation of the energy flow (through the work done by pressure forces) into the kinetic energy of vibration of a plate with an associated mass of fluid. In the nonlinear problem [with allowance for the term $\sim A^4$ in (3.2)], potential energy becomes positive with an increase in A_1. This determines the limit of instability, since no further increase in A_1 is possible after potential energy reaches the value equal to a const in (3.2).

The above energy interpretation of the onset of static instability can be generalized to the case of an arbitrary number of coupled plates. It is also applicable to a wide range of conservative nonequilibrium systems and to wave fields in conservative nonequilibrium media. Following [16], we will refer to the instabilities that develop in such systems (media) as reactive. They correspond to complex-conjugate roots of a characteristic (dis-

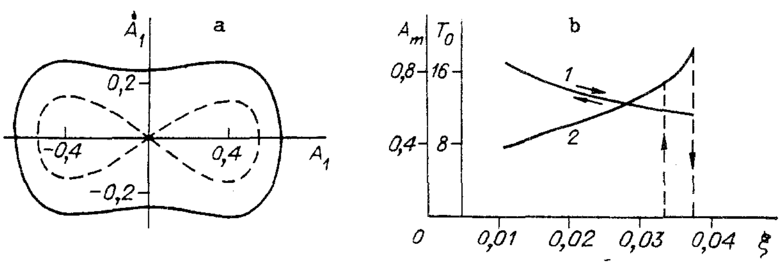

Fig. 2

persion) equation with real coefficients. Generalizing the above results, we see that the existence of reactive instability should be linked with the absence of a definite sign for energy as the quadratic form of the generalized coordinates and velocities (momenta) forming the phase space of the system.* It is in this case that infinite isoenergetic surfaces containing separatrices and other infinite phase trajectories appear in the phase space.

Divergence of the pair of plates at the parameter values indicated in Part 1 occurs at $V > V_{c1} \approx 1.48$. Meanwhile, the characteristic equation has one imaginary root in the region $V_{c1} < V < V_{c2} \approx 1.85$ and two such roots at $V > V_{c2}$ [6]. Thus, complex behavior by the phase trajectories is seen from the solution of the equations of the conservative problem for two coupled plates in the region of their divergence, with the initial conditions for $A_{1,2}$ and $\dot{A}_{1,2}$ being in the form of small "seeds". In this case, the trace of the phase trajectory on the planes A_1, $\dot{A}_1$ and A_2, $\dot{A}_2$ fills a region having the form of a "butterfly" which is symmetrical relative to the coordinate axes (the same result is obtained for the Duffing equation with a periodic external force — see [14], p. 55). We present the mean parameters for such randomness at $V = 2$: $\langle A_{1,2}\rangle \approx 0$, $A_{1,2*} = \langle A_{1,2} - \langle A_{1,2}\rangle\rangle^{1/2} \approx 0.4$, $K = \langle (A_1 - \langle A_1\rangle)(A_2 - \langle A_2\rangle)\rangle / A_{1*}A_{2*} \approx 0.69$ ($\langle \ldots \rangle$ denotes averaging over time τ). A minus sign for the correlation coefficient K indicates that the deflection of the plates is primarily antiphase in nature (the mechanism of this phenomenon was discussed in [5]). As is known, one of the main criteria used for dynamic chaos is the presence of positive Lyapunov exponents. The largest of these exponents λ_m determines the increment of the exponential acceleration of phase trajectories [13, 14]. At $V = 2$, we found the value $\lambda_m \approx 0.067$ for the characteristic frequency of vibration $\bar{\omega} \sim 0.3$ (methods of calculating λ_m were discussed in [14, 15].

At $\bar{r}$, $\xi \ll 1$, $\beta = 0$, we obtain the following from (2.1) for the rate of change in the energy of an individual plate

$$\frac{d\tilde{H}}{dt} = -2\bar{r}\dot{A}_1^2 - \frac{\xi}{m_0^2}(d_s A_1 - 3A_1^3)^2 \leqslant 0, \tag{3.3}$$

where $\tilde{H} = H - (\xi/m_0)(d_s - 3A_1^2)\dot{A}_1 \approx H$. It follows from (3.1) that normal dissipation (losses) leads to a decrease in the "quasi-energy" $\tilde{H}$ until one of its local minima (coinciding with the local minimum of H) is reached. This marks the establishment of one of the two non-neutral equilibrium states of the plate.

It can be concluded from the above analysis that the character of reactive instability is not crucial to the onset of free vibration (nonlinear flutter). Flutter is possible with both oscillatory and quasi-static instability if the losses in the plates are compensated for by anomalous dissipation — an irreversible removal of energy from the flow.†

4. Nonlinear Flutter of Plates. Let us examine Eqs. (2.2-2.6) when the vibrations of the plates are intensified by the flow (β, $\beta_1 \neq 0$). The technique of averaging over nonlinear vibrations can be used with system (2.2) for one plate (see [12], for example). Here,

*In the case of a distributed system, we would be concerned with the phase space for a single Fourier solid harmonic of the wave field [17].

†The anomalous dissipation which takes place during supersonic panel flutter in a uniform flow is due to the radiation, into free space, of acoustic waves carrying negative energy.

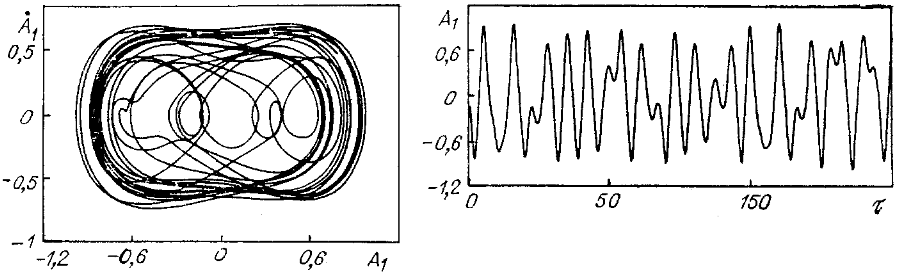

Fig. 3

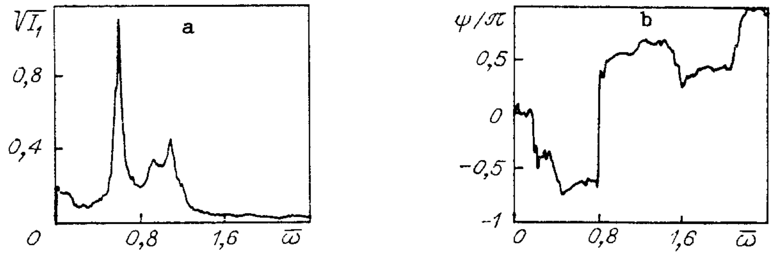

Fig. 4

$\bar{r}$, ξ, and $\beta \ll 1$ are used as small parameters, while the generating family of periodic solutions is determined by Eq. (3.1). The qualitative results to be obtained from such an approach can be predicted by using a frequency characteristic of the rate of energy intake during quasi-harmonic plate vibration $\chi(\bar{\omega})$. At $\xi = 0$, we obtain $\chi(\bar{\omega}) \sim \alpha_1 V^2 G_i(\bar{\omega}/V) - 2\bar{r}\bar{\omega}$. The frequency of vibration of nonlinear oscillatory (3.1) increases with an increase in the amplitude of these vibrations (at $d_s > 0$, this applies to periodic phase trajectories embracing the separatrix contour). When frequency lies within the region in which $\chi < 0$, the increase in amplitude ceases and a steady-state regime is established. As follows from (3.3), radiation losses can only lower the amplitude of free vibration.

These conclusions are supported by the results of numerical solution of (2.2) with the initial conditions $\zeta_1(0) = \dot{\zeta}_1(0) = 0$ and $|A_1(0)|$, $|\dot{A}_1(0)| \ll 1$. At $d_s > 0$, the limiting cycle in the plane A_1, $\dot{A}_1$ always embraces the separatrix contour of Eq. (3.1). Figure 2a shows such a limiting cycle obtained with $V = 2$, $\bar{r} = 0.004$, $\xi = 0.025$, and the enclosed separatrix contour of Eq. (3.1) (dashed curve). Figure 2b shows the dependence of the amplitude of the free vibrations A_m and their period T_0 (curves 1 and 2) on ξ ($V = 2$, $\bar{r} = 0.004$). It has the hysteresis loop typical of impulsively excited systems. The threshold value $\xi = \xi_1 \simeq 0.038$ at which the vibrations suddenly cease corresponds to the creation of a stable limiting cycle embracing the separatrix contour and a pair of unstable limiting cycles inside the separatrix loop (Fig. 2a). At the point $\xi = \xi_2 \simeq 0.033$ at which vibrations are suddenly excited, the unstable limiting cycles merge with focus-type equilibrium states and impart their instability to the latter. The dependence of the amplitude and period of the vibrations on $\bar{r}$ with fixed ξ is similar in form. Nonimpulsive excitation was observed at $V < V_c \simeq 1.65$.

The system of equations for a pair of plates was solved with the initial conditions $\zeta_{1,2}(0)$, $\dot{\zeta}_{1,2}(0)$, $\eta(0)$, $\dot{\eta}(0) = 0$; $|A_{1,2}(0)|$, $|\dot{A}_{1,2}(0)| \ll 1$. The existence of a conservative system of a nonattracting stochastic set in the phase state (see Part 3) results in the appearance of a random attractor in the phase state of weakly dissipative system (2.3-2.6). Figures 3 and 4 show results of calculation of the main characteristics of random steady-state vibrations at $V = 2$, $\bar{r} = 0.004$, and $\xi = 0.01$. The deflection spectrum of the first plate I_1 and the corresponding spectrum of the pair of plates Γ were calculated from the formulas

$$I_1 = \frac{1}{2\pi T}\langle |A_{1\bar{\omega}}|^2\rangle, \quad \Gamma = \frac{1}{2\pi T}\langle A^*_{2\bar{\omega}} A_{1\bar{\omega}}\rangle, \quad A_{1,2\bar{\omega}} = \int_0^T \tilde{A}_{1,2}(\tau)\, e^{i\bar{\omega}\tau} d\tau, \quad \tilde{A}_{1,2} = A_{1,2} - \frac{1}{T}\int_0^T A_{1,2} d\tau,$$

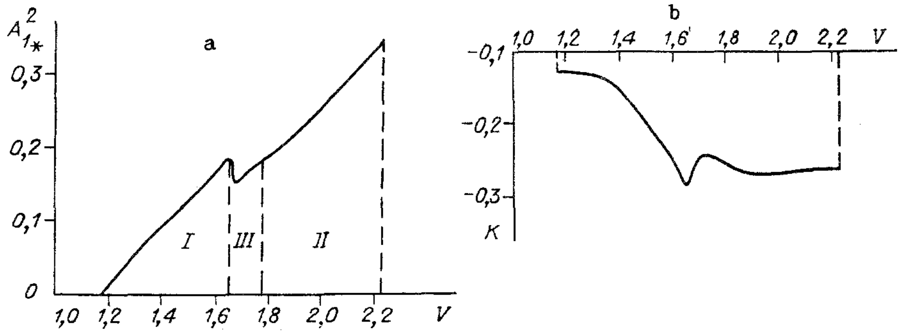

Fig. 5

where T is the length of the realization in terms of τ; <...> denotes averaging over the ensemble (in the given case, we averaged 60 realizations of the length T = 400). The dependence of $|\Gamma|^{1/2}$ on $\bar{\omega}$ is similar to that for $I_1^{1/2}$ shown in Fig. 4a. The phase characteristic of the reciprocal spectrum $\psi = \arg \Gamma$ is shown in Fig. 4b. In calculating the main Lyapunov exponent for the given regime, we obtained the value $\lambda_m \approx 0.12$.

Figure 5 shows the dependence of the rate of oscillation of deflection A^2_{1*} (a) and the correlation coefficient (b) on V at $\bar{r} = 0.004$ and $\xi = 0.01$. The mean displacements of the plates were close to zero for all values of V, with the relation $A_{1*} \approx A_{2*}$ being satisfied. Strictly periodic natural vibrations were generated in region I. These vibrations were similar to those described above for one plate (Fig. 2a). A random region was established in region II. The transition from regular to random vibrations occurred within a relatively narrow region III. Impulsive excitation of random free vibrations was observed in II (similar to the result shown in Fig. 2b).

The data shown in Figs. 3-5 makes it possible to explain the origin of dynamic chaos in the system we are considering. Independent self-excited plate-oscillators are synchronized in velocity region I in Fig. 5 when they are coupled, while they become unsynchronized in velocity region II due to phase mixing which occurs under the influence of conservative constraints. Here, a quasi-periodic vibration corresponding to the sharp peak in the frequency spectrum in Fig. 4a is generated. The incidence of the phase trajectory inside the separatrix contour of an individual oscillator (Figs. 2a and 4a) represents failures of the periodic regime, resulting in a diffuse high-frequency peak in the power spectrum. In accordance with Fig. 4b, the phase shift of the oscillations of plate deflection in these two main peaks of the frequency spectrum is displaced slightly from the values $\pm\pi/2$ in the direction of antiphase deflection $\pm\pi$. This accounts for the negative value of the total correlation coefficient. The disappearance of free vibrations with an increase in V (Fig. 5) can be attributed to the effect of nonlinear absorption, which prevents an increase in their amplitude. Our calculations showed that the oscillatory regime does not disappear with an increase in V at $\xi = 0$. A disruption in vibration similar to that just mentioned also occurs with an increase in V for independent plate-oscillators. This development is connected with expansion of the loop of the separatrix contour along the A_1 axis with an increase in V. When $\xi \neq 0$, the limiting cycles on the phase plane of the uncoupled oscillators approach this contour and eventually disappear. No free vibrations are excited in this case in the system of coupled plates. It should be kept in mind that the vibrations stop at values of V located at the limit of applicability of the unimodal approximation for plate deflection.

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OPTIMUM WING SHAPES IN A HYPERSONIC NONEQUILIBRIUM FLOW

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UDC 533.6.011.55

The trajectories of aerospace vehicles include sections of hypersonic flight at an angle of attack characterized by substantial nonequilibrium flow over the bottom surface of the wing (lifting body) [1, 2]. The thin-shock-layer method has proven very fruitful for general study of the effect of nonequilibrium physicochemical processes on the flow field and the aerodynamic characteristics of wings. Using this approach for a gas of variable density, Stalker [3] generalized well-known solutions for a delta wing in the case of non-Newtonian flow. The ideas set forth in [4, 5] were used in [6, 7] to integrate a system of equations for a three-dimensional nonequilibrium shock layer on a low-aspect-ratio wing of arbitrary form. An effective method was proposed in [8] for solving the two-dimensional system of equations obtained by integration for the form of the surface of the lead shock. Proceeding on the basis of the analytical solution in [4], the authors of [9] formulated a variational problem involving determination of the configuration of a wing with optimum aerodynamic performance. Despite the fact that the flow field around the wing is three-dimensional — in contrast to the cases examined in [10] — the solution reduces to the minimization of a unidimensional functional. The results in [9] pertain to the limiting cases of a wing in a flow of an ideal gas or equilibrium reacting air.

In the present study, we propose a variational method of determining the shape of a wing which is to perform optimally under hypersonic conditions for the general case of chemically nonequilibrium flow. The solutions that are obtained reveal features of the design that allow an improvement in the aerodynamic performance of wings and lifting bodies in a relaxing hypersonic flow.

1. A highly approximate estimate for pressure is obtained from the limiting Newtonian scheme of hypersonic flow with an infinitesimally thin shock layer on the surface of a body and the density ratio on the coincident lead shock $\rho_\infty/\rho_s^0 = \varepsilon = 0$. Examining the next approximation, with small nontrivial values of ε, makes it possible to more accurately evaluate

Zhukovskii. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 2, pp. 66-72, March-April, 1993. Original article submitted February 7, 1992.

 0021-8944/93/3402-0212$12.50